MICROWAVE IMAGING OF TWO-DIMENSIONAL CONDUCTING SCATTERERS USING PARTICLE SWARM OPTIMIZATION

Ioannis T. Rekanos and Maria A. Kanaki

Department of Informatics and Communications, Technological and Educational Institute of Serres, Terma Magnesias, GR-62124, Serres, Greece

Abstract – In this paper, a microwave imaging technique for reconstructing the shape of two-dimensional perfectly conducting scatterers by means of the particle swarm optimization algorithm is proposed. The reconstruction is based on scattered field simulated measurements derived by transverse magnetic illuminations. Two different implementations of the particle swarm optimization algorithm, namely the synchronous and the asynchronous one, are considered. Furthermore, the robustness of the algorithm with respect to different initial particle populations is investigated. Finally, the performance of the technique in the case of noisy scattered field data is examined.

Introduction

Microwave imaging is related to the reconstruction of the electromagnetic properties of unknown scatterers and it is of significant interest due to its numerous applications, such as biomedical imaging, nondestructive testing, and geophysical exploration. This inverse scattering problem is nonlinear and ill-posed and it is usually solved by means of iterative optimization algorithms and regularization schemes [1]. A particular problem of this kind is the estimation of the location as well as the shape of perfectly conducting scatterers [2], [3]. Previously, deterministic techniques [2] have been utilized to cope with the reconstruction of 2D conducting scatterers, while stochastic algorithms like the differential evolution algorithm [3] have been proposed.

Recently, the particle swarm optimization (PSO) algorithm [4] has attracted the interest of the electromagnetics community as a promising technique for the solution of optimization and design electromagnetic problems [5-6]. Actually, the PSO algorithm has already been applied to the solution of inverse scattering problems related to the reconstruction of dielectric scatterers [7-8].

In this paper, the PSO is applied to the reconstruction of the shape of perfectly conducting scatterers. The objective of the PSO is to minimize a cost function that describes the discrepancy between the measured and estimated scattered field data. In particular, the estimation of the scattered field data is based on the integral formulation of the direct scattering problem, while the contour of the scatterer is described by a parametric model utilizing cubic B-splines [9].

Description of the Problem

Our goal is to reconstruct the contour of the 2D perfectly conducting scatterer, given a set of monochromatic transverse magnetic (TM) scattered field measurements. Concerning the direct scattering problem, if \( C \) is the contour of the scatterer, then the scattered field at any position \( r \), on the surface or outside the scatterer domain, is given by the closed line integral
where $J(r')$ is the equivalent surface current density. Since the total field on the surface vanishes, $J(r')$ is derived by the solution of the integral equation

$$E'(r) = \frac{\mu_0}{4} \oint_{C} J(r') H_0^{(2)}(k_0 |r - r'|)dr'$$

where $E'(r)$ is the known TM incident field. Given the incident field, in order to compute the scattered field, we solve (2) with respect to $J(r')$ and then we substitute the solution into (1). The equivalent surface current density can be computed by applying the method of moments [10] to the integral (2).

Cubic B-Spline Representation of the Scatterer

The contour of the scatterer is described by a parametric model utilizing closed cubic B-splines [9] with $N$ control parameters $p_0, p_1, ..., p_{N-1}$. According to this approach, the contour is assumed smooth, since its second derivative is continuous, while its third derivative is piecewise continuous. In particular, the contour is written as

$$C(\theta) = R \left( \frac{N}{2\pi} \theta \right), \quad \theta \in [0, 2\pi]$$

where the curve $R(\tau), \quad (0 \leq \tau \leq N)$ is composed of $N$ curve segments $r_n(\tau)$, i.e.,

$$R(\tau) = \sum_{n=0}^{N-1} r_n(\tau - n).$$

Each curve segment $r_n(\tau)$ is a linear combination of four cubic polynomials defined in the normalized domain $t \in [0,1]$, i.e.,

$$r_n(t) = p_{n-1}Q_0(t) + p_nQ_1(t) + p_{n+1}Q_2(t) + p_{n+2}Q_3(t), \quad n = 0, 1, ..., N-1$$

where $p_{-1} = p_{N+1}$, $p_0 = p_N$, $p_1 = p_{N-1}$, while the cubic polynomials are given by

$$Q_0(t) = \frac{1}{6}(1-t)^3, \quad Q_1(t) = \frac{1}{2}t^3 - t^2 + \frac{2}{3}, \quad Q_2(t) = -\frac{1}{2}t^3 + \frac{1}{2}t^2 + \frac{1}{2}t + \frac{1}{6}, \quad Q_3(t) = \frac{1}{6}t^3.$$

Therefore, the contour $C$ is described by the $N$-dimensional vector, $\mathbf{p} = [p_0, p_1, ..., p_{N-1}]$, of the control parameters associated with the B-spline representation of $C$.

A significant advantage of the cubic B-spline representation of the scatterer contour is that the cubic polynomials are nonnegative. Thus, the reasonable constraint $C(\theta) \geq 0$ can be simply imposed by requiring all control parameters $p_n$ to be positive. Actually, this approach of imposing the constraint $C(\theta) \geq 0$ is based on the property

$$\min_{0 \leq \tau \leq N-1} p_n \leq R(\tau) \leq \max_{0 \leq \tau \leq N-1} p_n, \quad 0 \leq \tau \leq N.$$
\[ F(C) = \left[ \sum_{i=1}^{I} \sum_{m=1}^{M} \left( E_{im}^d - E_{im}^e(C) \right)^2 \right]^{1/2} = \left[ \sum_{i=1}^{I} \sum_{m=1}^{M} \left( E_{im}^d \right)^2 \right]^{1/2} \] 

where \( I \) is the total number of incidences, \( M \) is the number of measurements per incidence, \( E_d^m \) denotes the measured scattered field and \( E^e \) is the estimated one. Actually, since the contour is a function of \( p = [p_0, p_1, ..., p_{Npp}] \), (7) is minimized with respect to \( p \).

**Particle Swarm Optimization Algorithm**

In this study, the minimization of the cost function (7) is carried out by the PSO algorithm. Initially, a randomly generated set of \( K \) candidate solutions \{\( p_i : k = 1, 2, ..., K \}\} compose the swarm, where each solution is a position in the \( N \)-dimensional solution space. Moreover, each particle \( p_k \) has its own velocity \( v_k \), which is also initially randomly selected. The particles of the swarm are allowed to travel in the solution space searching for better solution positions in terms of their performance with respect to the cost function. A particular position of a particle is considered better that another if the corresponding value of the cost function is lower. Each particle remembers its best position \( f_k \) ever visited, while the best position \( g \) among all particles, namely the global best, is communicated to all particles.

After the initialization step of the PSO, the algorithm enters an iterative process where all particles update their velocities and positions. In particular, during each iteration of the PSO, the velocities and the positions are updated according to the scheme

\[ v_{kn}^{new} = A \cdot v_{kn}^{old} + B_1 \cdot r_1 \cdot (f_{kn} - p_{kn}^{old}) + B_2 \cdot r_2 \cdot (g_n - p_{kn}^{old}) \]  

(8)

\[ p_{kn}^{new} = p_{kn}^{old} + v_{kn}^{new} \]  

(9)

where \( 1 \leq k \leq K \) and \( 1 \leq n \leq N \). In (8), \( A \) is the inertia, \( B_1 \) the cognitive, and \( B_2 \) the social parameter, while \( r_1 \) and \( r_2 \) are random numbers uniformly distributed between 0 and 1. The inertia parameter \( A \) describes the tendency of the particle to travel along the same direction it has been traveling [11]. A large inertia value allows wide range search in the solution space, while a small one facilitates local exploration. It is generally accepted that a large inertia is suitable for the beginning of the PSO algorithm, while its value should slowly decrease as the iterations of the algorithm proceed. The cognitive parameter \( B_1 \) controls the attraction of a particle toward the best position it has ever visited, while the social parameter \( B_2 \) tunes the attraction of the particles toward the global best position [11]. According to the updating scheme described by (8) and (9) it is possible for all particles in the swarm to profit from their individual as well as the swarm community discoveries about the solution space. Finally, if the new position of a particle is better than its best position ever visited, then its current position is considered as the individual best one.

It has to be mentioned that the positions updating scheme (9) may result in candidate solutions that are not feasible. For example (9) may give negative values for the control parameters of the cubic B-spline representation. In such a case, the particle position is corrected by placing it back into the admissible solution space. This can be done by applying the “reflecting wall” boundary condition [5].

There are two basic approaches in the way the global best position \( g \) is communicated to all particles, namely the synchronous and the asynchronous mode. According to the synchronous mode, the global best is found and communicated after the updating procedure has been carried out for all particles and their performance with respect to the cost function has been evaluated. In the asynchronous mode the global best position is evaluated and communicated whenever a particle has reached a better position compared to the current global best. In general, the asynchronous mode may speedup the convergence of the PSO, since it is more elitist. However, the synchronous mode may allow the search of wider regions of the solution space.
Numerical Results

As a first application example of microwave imaging by means of the PSO algorithm, we have considered the reconstruction of the contour of a single 2D perfectly conducting scatterer. The scatterer, whose original contour is shown in Fig. 1, is illuminated by 7 TM plane wave incidences, uniformly distributed. The maximum radius of this particular scatterer is approximately $1.5\lambda$, where $\lambda$ is the wavelength of the monochromatic incidences. For each incidence, the scattered field is measured at 32 positions, uniformly placed around the scatterer at a distance from the scatterer center equal to $10\lambda$. These measurements were simulated by means of the method of moments. For the reconstruction of the scatterer shape, the contour has been represented by $N=10$ control parameters of the cubic B-spline model.

The population size, $K$, of the particle swarm was selected equal to 40. The initial values of the components of the particles that correspond to the control parameters were chosen randomly (uniformly distributed in the range $[0, 2]$). The initial values of the components of the particle velocities were also chosen randomly (uniformly distributed in the range $[-1, 1]$). Then, 200 iterations of the PSO algorithm have been completed, following the synchronous mode implementation. For all iterations, the cognitive and the social parameter were kept constant ($B_1 = B_2 = 0.5$) [11], whereas the inertia parameter was initially $\omega = 0.9$ and dropped linearly to 0.7 after 200 iterations. The value of the cost function after the completion of the optimization process was lower than $5.8 \times 10^{-3}$. The reconstruction result is shown in Fig. 1, where the original shape of the scatterer and the finally reconstructed one are illustrated. In addition, Fig. 1 depicts the best contour estimate, which was included in the randomly generated initial swarm.

![Fig.1. Shape reconstruction result. Original contour (circles), best estimate included in the initial swarm (diamonds), best estimate after 200 iterations (crosses).](image)

To quantify the accuracy of the shape reconstruction, we define the reconstruction error given by

$$E(C) = \left[ \frac{1}{L} \sum_{l=1}^{L} \left[ C^d(\theta_l) - C^e(\theta_l) \right]^2 \right]^{1/2}$$

where $C^d(\theta)$ and $C^e(\theta)$ stand for the original and the estimated contour (radius function) of the scatterer. In (10), $L$ is the total number of distinct points along the contours, where the original and
the estimated radius are evaluated. Actually, the corresponding angular coordinates of the distinct point are \( \theta_l = (2\pi / L)l \) with \( l = 1, 2, \ldots, L \). In the application examples, \( L \) was selected equal to 60. The reconstruction error after 200 iterations was lower than \( 2.9 \times 10^{-2} \). The cost function and the reconstruction error versus number of iterations are illustrated in Fig. 2.

![Graph](image)

**Fig. 2.** Results related to the reconstruction of the scatterer shown in Fig. 2. Cost function (a) and reconstruction error (b) versus number of iterations of the PSO algorithm.

In the second application example of the PSO algorithm, we compare the synchronous and the asynchronous mode of implementation. We consider 30 different realizations of perfectly conducting scatterers. Each scatterer has been reconstructed by applying both the synchronous and the asynchronous mode of the PSO. All the parameters related to the second application were set as in the first application described above. Along the 30 realizations examined, the minimum, the maximum, and the mean values of the cost function and the reconstruction error, after 200 iterations of the PSO algorithm (for the case of the synchronous and asynchronous mode) are shown in Table I. It is clear that the asynchronous implementation of the PSO outperforms the synchronous one. However, it should be noted that a parallel implementation of the asynchronous mode is not advantageous. From Table I we can also conclude that the PSO is efficient in reconstructing the scatterer contour for various particle swarm initializations.

**TABLE I**

<table>
<thead>
<tr>
<th>PSO Mode</th>
<th>Cost Function, ( F(C) )</th>
<th>Reconstruction Error, ( E(C) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>Synchronous</td>
<td>( 3.7 \times 10^{-3} )</td>
<td>( 2.0 \times 10^{-2} )</td>
</tr>
<tr>
<td>Asynchronous</td>
<td>( 2.1 \times 10^{-8} )</td>
<td>( 1.7 \times 10^{-2} )</td>
</tr>
</tbody>
</table>

Finally, the robustness of the PSO in the presence of noisy scattered field measurements has been examined. In particular, a circular perfectly conducting scatterer, with radius equal to \( \lambda \), has been considered and reconstructed following the same measurement and PSO setup as in the first application. The measurements have been corrupted by additive white Gaussian noise, while the performance of the algorithm for different signal-to-noise ratio (SNR) levels has been tested. The SNR level in dB is given by
where $\sigma^2$ is the variance of the Gaussian noise. In Table II, the cost function and the reconstruction error after 200 iterations of the asynchronous PSO algorithm are shown, for different SNR levels. From Table II, we can conclude that the PSO results in accurate reconstruction ($E(C) < 5.0 \times 10^{-3}$) even for SNR level equal to 15dB.

TABLE II

<table>
<thead>
<tr>
<th>SNR (dB)</th>
<th>$\infty$</th>
<th>25</th>
<th>20</th>
<th>15</th>
<th>10</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F(C)$</td>
<td>4.27 $\times 10^{-7}$</td>
<td>1.08 $\times 10^{-4}$</td>
<td>9.03 $\times 10^{-3}$</td>
<td>2.69 $\times 10^{-2}$</td>
<td>9.35 $\times 10^{-2}$</td>
<td>2.45 $\times 10^{-1}$</td>
</tr>
<tr>
<td>$E(C)$</td>
<td>1.05 $\times 10^{-4}$</td>
<td>8.42 $\times 10^{-3}$</td>
<td>2.45 $\times 10^{-3}$</td>
<td>3.53 $\times 10^{-3}$</td>
<td>7.83 $\times 10^{-3}$</td>
<td>1.51 $\times 10^{-2}$</td>
</tr>
</tbody>
</table>

Conclusions

A microwave imaging technique based on the PSO algorithm has been proposed for the reconstruction of the shape of 2D perfectly conducting scatterers. Numerical results show that the PSO is an efficient and very promising algorithm for microwave imaging applications. Both the synchronous and the asynchronous modes of the PSO have been examined. The asynchronous mode seems to give more accurate results (for the same number of iterations) compared to the synchronous one. Furthermore, the PSO algorithm is robust in the presence of noisy scattered field measurements.

Acknowledgement

This work has been supported by the Greek Ministry of Education under the “Archimedes” Project of the Technological and Educational Institute of Serres.

References